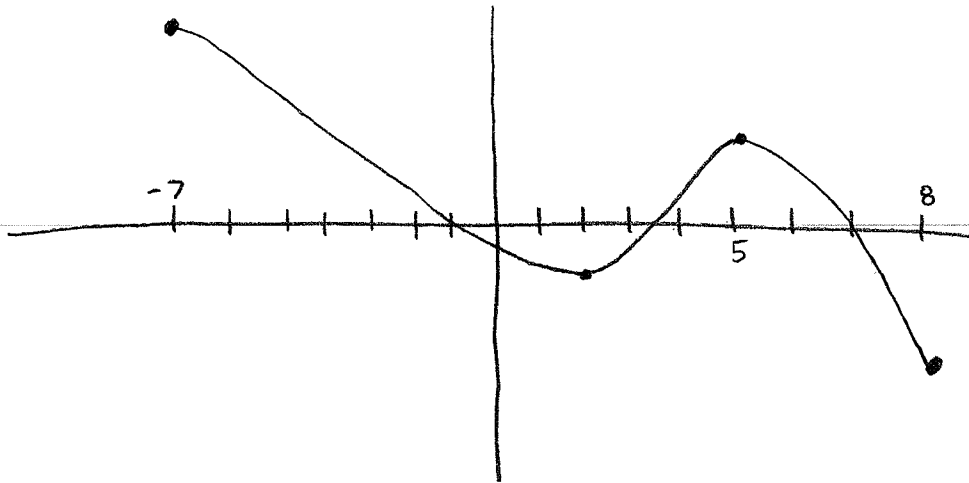


1. Draw the graph of a function that meets the following conditions:

- a. domain  $[-7, 8]$
- b.  $f'(2) = 0$
- c.  $f'(x) < 0$  for  $-7 < x < 2$
- d.  $f'(x) > 0$  for  $2 < x < 5$
- e.  $f'(5) = 0$
- f.  $f'(x) < 0$  for  $5 < x < 8$
- g. absolute maximum at  $x = -7$
- h. absolute minimum at  $x = 8$
- i. relative maximum at  $x = 5$



2. Find the intervals on which  $g(x)$  is increasing or decreasing. Find the coordinates of all relative extrema. Justify all answers analytically.

$$g(x) = \frac{x^2 - 2x + 1}{x + 1}$$

$$g'(x) = \frac{(2x - 2)(x + 1) - (x^2 - 2x + 1)(1)}{(x + 1)^2}$$

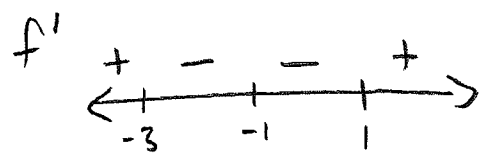
$$g'(x) = \frac{2x^2 - 2 - x^2 + 2x - 1}{(x + 1)^2}$$

$$0 = \frac{x^2 + 2x - 3}{(x + 1)^2}$$

$$x^2 + 2x - 3 = 0 \quad (x + 1)^2 = 0$$

$$(x + 3)(x - 1) = 0 \quad x = -1$$

$$x = -3, 1$$



$$f'(-4) = \frac{16 - 8 - 3}{+} = \frac{+}{+}$$

$$f'(-2) = \frac{4 - 4 - 3}{+} = \frac{-}{+}$$

$$f'(0) = \frac{-}{+} = -$$

$$f'(2) = \frac{4 + 4 - 3}{+} = \frac{+}{+}$$

$f$  is increasing  $(-\infty, -3) \cup (1, \infty)$   
because  $f' > 0$  on the intervals  
 $f$  is decreasing  $(-3, -1) \cup (-1, 1)$   
because  $f' < 0$  on the intervals

REL MAX  $(-3, -8)$  because  $f'$  changes from pos to neg at  $x = -3$   
 $f(-3) = \frac{9 + 6 + 1}{-8} = -8$   
 REL MIN  $(1, 0)$  because  $f'$  changes from neg to pos at  $x = 1$

3. Discuss the concavity. Give the coordinates of any inflection points. Justify all answers analytically.

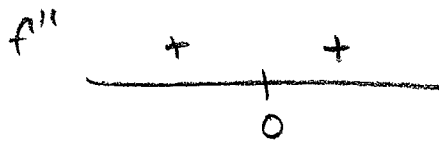
$$f(x) = 2x^4 - 8x + 3$$

$$f'(x) = 8x^3 - 8$$

$$f''(x) = 24x^2$$

$$24x^2 = 0$$

$$x = 0$$



$$f''(-1) = +$$

$$f''(1) = +$$

$f$  is conc up  $(-\infty, 0) \cup (0, \infty)$   
 because  $f'' > 0$  on the intervals

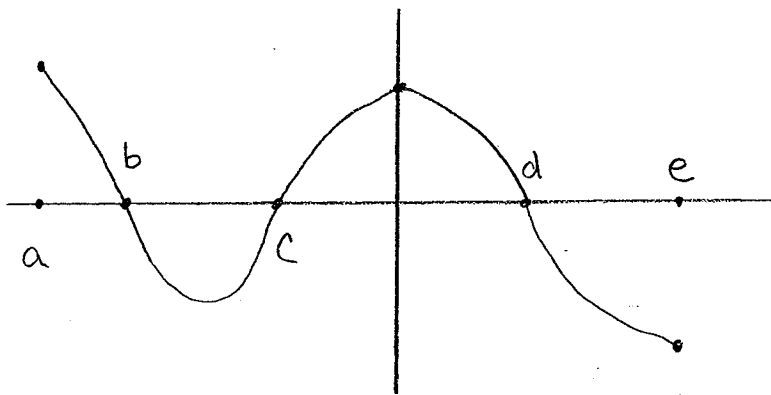
NO INFL PTS BECAUSE

$f''$  NEVER CHANGES

FROM POS TO NEG OR

NEG TO POS

4. Below is a graph of  $f''(x)$ . Answer the following questions.



- a. On what intervals is  $f$  concave down?

$$(b, c) \cup (d, e) \text{ b/c } f'' < 0$$

- b. On what intervals is  $f'$  decreasing?

$$(b, c) \cup (d, e) \text{ b/c } f'' < 0$$

- c. What are the critical numbers of  $f'$ ?

$$x = b, c, \text{ and } d \text{ because } f'' = 0$$

at these  $x$ -values