AP Calculus

Name Key 10/21/2011

Chap 2 Review

1. Use the definition of the derivative to find f ' (x).

$$f(x) = 2x^2 + x - 1$$

$$\lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 + (x+\Delta x) - 1 - (2x^2 + x - 1)}{\Delta x}$$

$$\frac{2(x^2 + 2x\Delta x + \Delta x^2) + x + \Delta x - 1 - 2x^2 - x + 1}{\Delta x}$$

$$\frac{\cancel{2x^2} + 4x\Delta x + 2\Delta x^2 + \cancel{x} + \Delta x \cancel{-1} \cancel{-2x^2} \cancel{-x} \cancel{+1}}{\Delta x}$$

$$\Rightarrow \frac{\cancel{\Delta x}(4x + 2\Delta x + 1)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 4x + 2\Delta x + 1$$

$$= 4x + 1$$

2. Find the derivative.

$$y = \frac{x^3 + 3x + 2}{x^2 - 1}$$

$$y' = \frac{(3x^2+3)(x^2-1) - (x^3+3x+2)(2x)}{(x^2-1)^2}$$

3. The velocity of an object is $v(t) = \sqrt{t}(36 - t^2)$ meters per second for $0 \le t \le 6$. Find the velocity and acceleration of the object when t = 4 seconds. What do the signs of the two quantities represent?

$$v(4) = \sqrt{4}(36 - 4^2)$$

$$= 2(20) = \boxed{40 \text{ m/s}}$$

$$a(t) = v'(t) = \tfrac{1}{2}t^{-\frac{1}{2}}(36 - t^2) + \sqrt{t}(-2t)$$

$$\boxed{v'(4) = -11 \text{ m/s}^2}$$

accel.

4. Find $\frac{dy}{dx}$. $y = 2\cot^2(\pi x + 2)$

$$y' = 2 \cdot 2\cot(\pi x + 2) \cdot (-\csc^2(\pi x + 2)) \cdot \pi$$

5. Find $\frac{dy}{dx}$ and evaluate at (2, 1).

$$x^3 + y^3 = 4xy + 1$$

$$3x^2 + 3y^2 \frac{dy}{dx} = (4)(y) + (4x)\left(\frac{dy}{dx}\right) + 0$$

$$-4x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 4y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{4y - 3x^2}{-4x + 3y^2}}$$

$$\frac{dy}{dx}_{(2,1)} = \boxed{8/5}$$

6. A spherical balloon is filling with air at a rate of $16\, ft^3/s$. Find the radius when $\frac{dr}{dt} = 1.5\, ft/s$.

Round to the nearest hundredth.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$16 = \frac{4}{3}\pi \cdot 3r^2 (1.5)$$

$$\boxed{r \approx .921\ ft}$$

Chap 2 Review-Part 2

1. Let f be the function $f(x)=\begin{cases} 2x^3+1 & x\le -3 \\ ax^2 & x>-3 \end{cases}$

For what value of *a* will *f* be continuous?

$2(-3)^3+1=a(-3)^2$

$-54+1=9a$

$\boxed{\frac{-53}{9}=a}$

2. At what point(s) does the graph of the function $f(x)=\dfrac{x-4}{x^2-7}$ have a horizontal tangent line?

$f'(x)=\dfrac{(1)(x^2-7)-(x-4)(2x)}{(x^2-7)^2}$

$x^2-7-2x^2+8x=0$

$-x^2+8x-7=0$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=7,1$

$f(7)=\dfrac{7-4}{49-7}=\dfrac{3}{42}=\dfrac{1}{14}$

$f(1)=\frac{-3}{-6}=1/2$

$\boxed{(7,1/14) \quad (1,1/2)}$

3. Find $\dfrac{d^2y}{dx^2}$ $\qquad y=2\sec x$

$y'=(2\sec x)\tan x$

$y''=(2\sec x\tan x)(\tan x)+(2\sec x)(\sec^2 x)$

$=\boxed{2\sec x\cdot\tan^2 x+2\sec^3 x}$

4. Find the equation of the tangent line to the graph of $f(x) = \frac{x}{x-1}$

that passes through the point (3, 3/2).

$$f'(x) = \frac{(1)(x-1) - (x)(1)}{(x-1)^2}$$

$$f'(x) = \frac{-1}{(x-1)^2}$$

$$f'(3) = \frac{-1}{4} = m$$

$$y - 3/2 = \frac{-1}{4}(x-3)$$

$$y - 3/2 = -1/4\,x + 3/4$$

$$+6/4$$

$$\boxed{y = -1/4\,x + 9/4}$$

5. If h(x) = f(x)*g(x), find h '(-2) given that f(-2)=3, f '(-2)=1, g(-2)=4, and g '(-2)=-3.

PR

$$h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$$

$$h'(-2) = (1)(4) + (3)(-3)$$

$$= 4 - 9$$

$$= \boxed{-5}$$

6. Using your graphing calculator, find f '(-3) if $f(x) = \sqrt[3]{x(2x^2 - 4x)}$

nderiv $(\sqrt[3]{\ }(x(2x^2-4x)), x, -3) \approx 1.295$

7. Find the point(s) on the graph of the function $y = 3x^4 - 2x^2 - 10$ where the slope of the tangent is 4.

$$y' = 12x^3 - 4x + 0$$

$$\frac{12x^3}{4} - \frac{4x}{4} = \frac{4}{4}$$

$$3x^3 - x = 1$$

$$3x^3 - x - 1 = 0$$

USE CALC TO FIND "ZERO" 4

$\Rightarrow x = .85138307$

$y = 3(.8514)^4 - 2(.8514)^2 - 10$

$y = -9.873$

$\boxed{(.8514, -9.873)}$

8. Find $\frac{dy}{dx}$ $\quad 2y^2 - xy + 3x^2 = 17$

$$4y\frac{dy}{dx} - \left((1)(y) + (x)\left(\frac{dy}{dx}\right)\right) + 6x = 0$$

$$4y\frac{dy}{dx} - y - x\frac{dy}{dx} + 6x = 0$$

$$\frac{dy}{dx}(4y - x) = -6x + y$$

$$\frac{dy}{dx} = \frac{-6x + y}{4y - x}$$