REVIEW SEMESTER 1

How is the derivative of a function $f(x)$ defined?

- Slope of a tangent line to a function
- instaneous rate of change
- local linearity

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Find the derivative of each of the following:

$$f(x) = (2 + \sin 3x)^4$$

$$\boxed{f'(x) = 4(2+\sin 3x)^3 \cdot \cos 3x \cdot 3}$$

$g(x) = 3x - 2x^2$ (use the limit definition)

$$\lim_{\Delta x \to 0} \frac{3(x+\Delta x) - 2(x+\Delta x)^2 - (3x - 2x^2)}{\Delta x}$$

$$\frac{\cancel{3x} + 3\Delta x - \cancel{2x^2} - 4x\Delta x - 2\Delta x^2 - \cancel{3x} + \cancel{2x^2}}{\Delta x}$$

$$\frac{\cancel{\Delta x}(3 - 4x - 2\Delta x)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 3 - 4x - 2\Delta x = \boxed{3 - 4x}$$

$$h(x) = \frac{2x}{5 + x^2}$$

$$h'(x) = \frac{(2)(5+x^2) - (2x)(2x)}{(5+x^2)^2}$$

$$h'(x) = \boxed{\frac{10 - 2x^2}{(5+x^2)^2}}$$

Find all intercepts for the function $y = -40 - 3x + x^2$

y-int $(0,-40)$

x-int $(8,0)$

$(-5,0)$

$-40 - 3x + x^2 = 0$

$x^2 - 3x - 40 = 0$

$(x-8)(x+5) = 0$

$x = 8, -5$

For $4x^2 - (2xy) = 3y$, find $\dfrac{dy}{dx}$.

$8x - \left(2y + 2x\frac{dy}{dx}\right) = 3\frac{dy}{dx}$

$8x - 2y - 2x\frac{dy}{dx} = 3\frac{dy}{dx}$

$8x - 2y = 3\frac{dy}{dx} + 2x\frac{dy}{dx}$

$\boxed{\dfrac{8x-2y}{3+2x} = \dfrac{dy}{dx}}$

For $4x^2 - 2xy = 3y$, find $\dfrac{d^2y}{dx^2}$.

Q.R.

$\dfrac{\left(8 - 2\frac{dy}{dx}\right)(3+2x) - (8x-2y)(2)}{(3+2x)^2}$

$\dfrac{8(3+2x) - 2\frac{dy}{dx}(3+2x) - 16x + 4y}{(3+2x)^2}$

$8(3+2x) - 2\left(\frac{8x-2y}{\cancel{3+2x}}\right)(\cancel{3+2x}) - 16x + 4y$

$24 + \cancel{16x} - \cancel{16x} + 4y - 16x + 4y$

$\Rightarrow$

$\boxed{\dfrac{24 - 16x + 8y}{(3+2x)^2} = \dfrac{d^2y}{dx^2}}$

Explain what it means if $f(x)$ is continuous at $x = a$.

(i) $\lim_{x \to a} f(x)$ exists

(ii) $f(a)$ exists

(iii) $\lim_{x \to a} f(x) = f(a)$

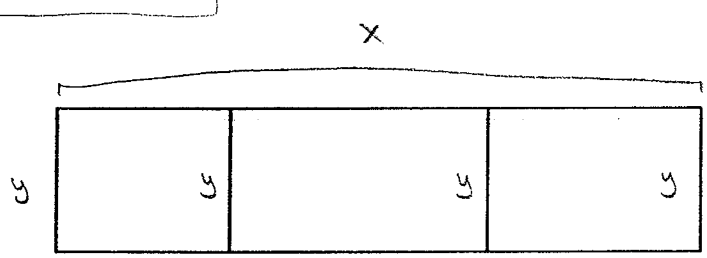
Discuss the continuity.

$$y = \frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}}{(\cancel{x-3})(x-4)} = \frac{1}{x-4} \qquad \begin{array}{l} x=3 \\ x=4 \end{array}$$

$\boxed{\text{Continuous } (-\infty, 3) \cup (3,4) \cup (4, \infty)}$

NR $x=4$

R $x=3$



A farmer has 800 meters of fencing that he plans to use to enclose 3 rectangular pens as shown above. What dimensions will give him the maximum area?

$2x + 4y = 800 \rightarrow \quad x = -2y + 400$

$A = xy$

$A = y(-2y + 400)$

$A = -2y^2 + 400y$

$A' = -4y + 400$

$0 = -4y + 400$

$4y = 400$

$y = 100$

$x = 200$

$\boxed{200\text{m} \times 100\text{m}}$

An open-top box is made with a square base and should have a volume of 6000 cubic inches. If the material for the sides costs \$0.20 per square inch and the material for the base costs \$0.30 per square inch, determine the dimensions of the box that minimize the cost of the materials.

Constraint (Volume) $x^2y = 6000 \text{ in}^3 \longrightarrow y = \frac{6000}{x^2}$

Objective (SA) $4xy(.2) + x^2(.3)$

$$4x\left(\frac{6000}{x^2}\right)(.2) + .3x^2 = C$$

$$\frac{4800}{x} + .3x^2 = C$$

$$-\frac{4800}{x^2} + .6x = C'$$

$$.6x = \frac{4800}{x^2}$$

$$.6x^3 = 4800$$

$$x = 20$$

$$y = 15$$

$$\boxed{20 \text{ in} \times 20 \text{ in} \times 15 \text{ in}}$$

Walt's Walnut Grove has determined that the annual yield per walnut tree is fairly constant at 50 pounds per tree when the number of trees per acre is 30 or fewer. For each additional tree over 30, the annual yield per tree decrease by 1.25 pounds due to overcrowding. How many trees should be planted on each acre to maximize the annual yield from an acre?

35 TREES

$$\begin{cases} 50t & \text{if } t \le 30 \\ t(50 - 1.25(t-30)) & \text{if } t \ge 30 \end{cases}$$

$$y = t(50 - 1.25(t-30))$$
$$= t(50 - 1.25t + 37.5)$$
$$= t(87.5 - 1.25t)$$
$$= 87.5t - 1.25t^2$$
$$y' = 87.5 - 2.5t = 0$$
$$87.5 = 2.5t$$
$$35 = t$$

$+ \quad -$

$t = 35$

MAX

$$\boxed{35 \text{ trees}}$$

Find the limits.

$$\lim_{x\to 3}\frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}}{(\cancel{x-3})(x-4)} = \lim_{x\to 3}\frac{1}{x-4} = \frac{1}{3-4} = \boxed{-1}$$

$$\lim_{x\to 4^-}\frac{x-3}{x^2-7x+12} = \lim_{x\to 4^-}\frac{1}{x-4} = \boxed{-\infty \quad \text{or DNE}}$$

$$\lim_{x\to 2} 2x-3 = 2(2)-3 = \boxed{1}$$

$$\lim_{x\to 2}\frac{5}{x-2} = \boxed{\text{DNE}}$$

$$\lim_{x\to 2^+}\sqrt{x-2} = \boxed{0}$$

$$\lim_{x\to 1^+}[x] = [1.001] = \boxed{1}$$

$$\lim_{x\to 1^-}[x] = [.999] = \boxed{0}$$

$$\lim_{x\to 1}[x] = \text{DNE} \qquad \lim_{x\to 1^+} \neq \lim_{x\to 1^-}$$

$$\lim_{x\to\infty}\frac{2}{x} = \boxed{0}$$

$$\lim_{x\to\infty}\frac{2x^2-4x}{x^2+1} = \frac{2}{1} = \boxed{2}$$

$$\lim_{x\to\infty}\frac{5x^2}{x-4} = \boxed{+\infty}$$

Find the equation of the tangent line of $f(x) = 4x - 2x^2$ at $x = 3$.

$f'(x) = 4 - 4x$

$f'(3) = 4 - 12 = -8 = m$

$f(3) = 12 - 2(9)$

$= -6$

$(3, -6) \quad m = -8$

$-6 = -8(3) + b$

$18 = b$

$\boxed{y = -8x + 18}$

The position of a particle moving along a straight line at any time t is given by $s(t) = t^3 - 2t + 4$. What is the acceleration of the particle when $t = 2$?

$s'(t) = v(t) = 3t^2 - 2$

$s''(t) = a(t) = 6t$

$a(2) = 6(2) = \boxed{12u/s^2}$

Find $f \circ g(x)$

$f(x) = \dfrac{1}{3 - x^2} \qquad g(x) = \sqrt{x - 1}$

$f(\sqrt{x-1}) = \dfrac{1}{3 - (\sqrt{x-1})^2} = \dfrac{1}{3 - x + 1} = \boxed{\dfrac{1}{4 - x}}$

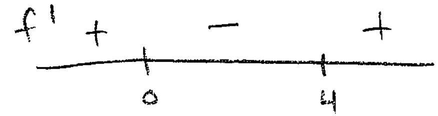
The function defined by $f(x) = x^3 - 6x^2 + 2$ for all real numbers x has a relative maximum for what value(s) of x?

$f'(x) = 3x^2 - 12x$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0 \quad x = 4$



$\boxed{\text{REL MAX at } \underline{x = 0} \text{ because } f' \text{ changes from pos to neg at } x = 0}$

$f'(-1) = 3 + 12$

$f'(1) = 3 - 12$

$f'(5) = 75 - 60$

Determine all value for x where $f(x) = x^4 - 4x^3 + 6x^2 + 4x + 1$ has an inflection point.

$f'(x) = 4x^3 - 12x^2 + 12x + 4$

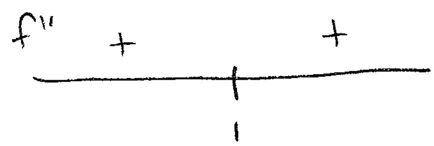
$f''(x) = 12x^2 - 24x + 12$

$12x^2 - 24x + 12 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$



$f''(0) = +$

$f''(2) = 48 - 48 + 12$

$\boxed{\text{NO INFL PTS}}$

Show an appropriate graph of $f(x)$ if $f'(x)$ is shown below.